Suppression of Quantum Fluctuations in a Josephson Junction Coupled to a Nanomechanical Resonator

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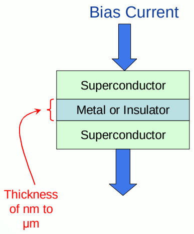
What is a Josephson Junction?

- 2 slabs of superconducting material, separated by a non-superconducting material.
- Metal/Insulator barrier acts as a superconductor when a bias current less than the junction's critical current I_c is applied.
- For a bias current greater than I_c, a voltage drop appears across the junction.

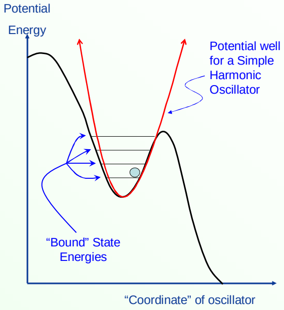
•We study junctions biased at currents less than I_c.



Oscillations in a Josephson Junction (JJ)

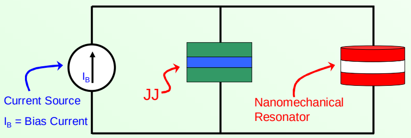
•Can think of it as a microscopic "particle" trapped in a potential well that is shaped like a tilted washboard.

•By approximating a well with a parabola, can treat the particle as a quantum simple harmonic oscillator.



System of Study

•We are interested in coupling our JJ in parallel with a nanomechanical resonator.



Why?

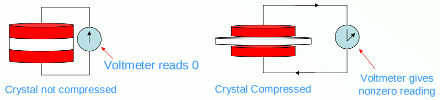
•It has been proposed that qubits (quantum bits) can be made from junction-resonator pairs.

•Quantum computers may one day be based on these devices in the same way that modern computers are based on transistors.

•Before using these devices for practical purposes, we must first learn more about their basic behavior, which is rich in physics.

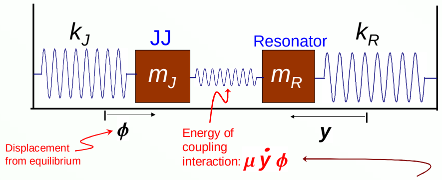
Nanomechanical Oscillator: AlN Dilatational Resonator

- Consists of a piezoelectric crystal sandwiched between two pieces of metal.
- The crystal produces a voltage when compressed, and vice versa.
- These devices are nanometers to micrometers in size and oscillate with frequencies of order of GHz.



JJ + Resonator = Coupled Quantum Oscillators

•Since the JJ is approximated as a harmonic oscillator, it can be treated like a block on a spring, coupled to the resonator, which is also a harmonic oscillator:



Cleland and Geller, Phys. Rev. Lett. **93**, 070501 (04).

Spring Constants: k_J, k_R

Natural frequencies: $\omega_J = \sqrt{\frac{k_J}{m_J}}$, $\omega_R = \sqrt{\frac{k_R}{m_R}}$

Effect of Resonator Coupling on Size of JJ Oscillations:

$$L_E = \underbrace{\frac{m_J}{2}\left(\frac{d\phi}{d\tau}\right)^2 + \frac{m_J}{2}\omega_J^2\phi^2 + \frac{m_R}{2}\left(\frac{dy}{d\tau}\right)^2 + \frac{m_R}{2}\omega_R^2 y^2 + i\,\mu\left(\frac{dy}{d\tau}\right)\phi}_{\text{in imaginary "time," } \tau \equiv i\,t}$$

Euclidean Lagrangian (K.E. + P.E)

Using the path integral approach we calculate the average square displacement of the Josephson "block/spring":

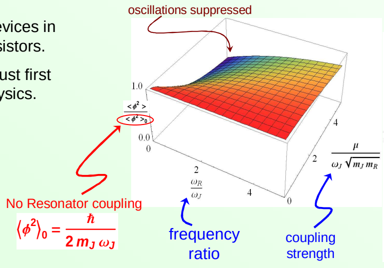
$$<\phi^2> = \sum_{n=-\infty}^{\infty} \frac{1}{\beta\left(m_J\left(\omega_n^2+\omega_J^2\right) + \frac{\mu^2\omega_n^2}{m_R\left(\omega_n^2+\omega_R^2\right)}\right)}$$

$$\beta = \frac{1}{k_B T} \qquad \omega_n = \frac{2\pi n}{\hbar\beta}$$

Boltzmann's Constant

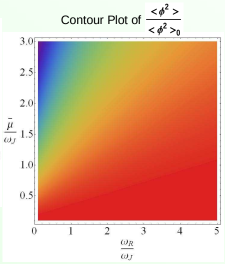
Fourier frequency, n is an integer

plotting the results:



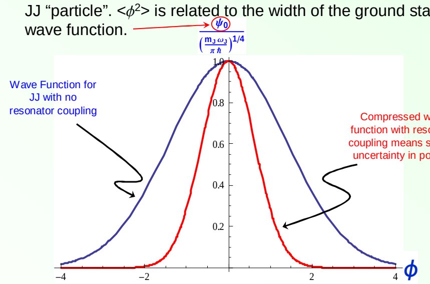
No Resonator coupling

$$\langle\phi^2\rangle_0 = \frac{\hbar}{2\,m_J\,\omega_J}$$



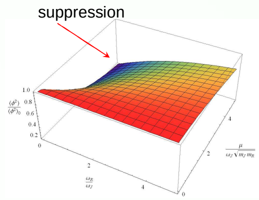
Effect of Resonator Coupling on Ground State Wave Function of JJ

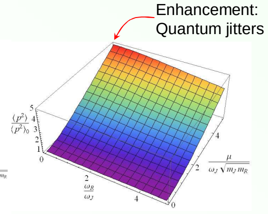
- $<\phi^2>$ is the square of the uncertainty in the position of our JJ "particle". $<\phi^2>$ is related to the width of the ground state wave function.



Heisenberg Uncertainty Principle

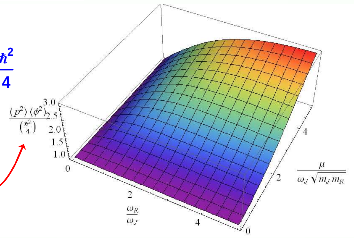
- Uncertainty in position is inversely related to the uncertainty in momentum: $\sigma_\phi\sigma_p \geq \frac{\hbar}{2}$
- We calculated $<p^2>$, the uncertainty in the momentum squared. Compare plots below:



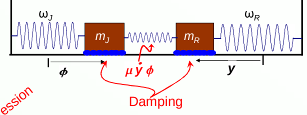


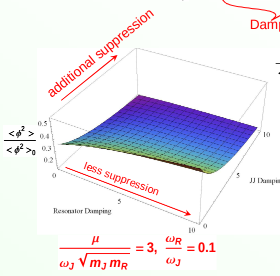
$$\langle p^2\rangle\langle\phi^2\rangle = \left(\sigma_\phi\sigma_p\right)^2 \geq \frac{\hbar^2}{4}$$

Plot of $<p^2>$ $<\phi^2>$ confirms Heisenberg Uncertainty principle for JJ coupled to a resonator.

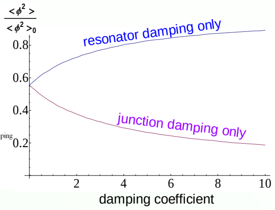


Adding Damping to the System





$$\frac{\mu}{\omega_J\sqrt{m_J m_R}} = 3, \quad \frac{\omega_R}{\omega_J} = 0.1$$

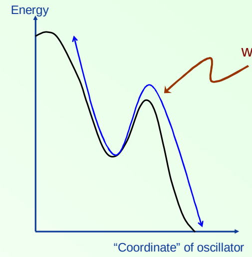


- Adding damping to the junction slows down the JJ particle, enhancing the suppression of $<\phi^2>$ due to coupling.
- Adding damping to the resonator slows down the resonator, which weakens coupling ($\mu\dot{y}\phi$), weakening the suppression of $<\phi^2>$ due to coupling.

Future Work

- Effect of anharmonic JJ potential energy on $<\phi^2>$:

$$V_{JJ} = \frac{1}{2}m_J\omega_J^2\phi^2\left(1-\frac{\phi}{\phi_1}\right)$$



Closer to tilted washboard potential than purely harmonic case

- Preliminary result:

$$\frac{\langle\phi^2\rangle}{\langle\phi^2\rangle_0} = \frac{\langle\phi^2\rangle_{SHO}}{\langle\phi^2\rangle_0}\underbrace{\left(1+\frac{9}{4}\frac{\langle\phi^2\rangle_{SHO}}{\phi_1^2}\right)}$$

Result for using harmonic form for V_{JJ}

Enhances quantum fluctuations in JJ, compared to harmonic case.

Conclusions

- We studied the behavior of a Josephson junction coupled in a parallel circuit to a nanomechanical resonator.
- Approximating the system as coupled harmonic oscillators, we found that junction oscillations are suppressed due to resonator coupling.
- We found that damping the junction enhances the suppression of the oscillations due to coupling. Adding damping to the resonator, however, weakens the suppression of junction oscillations.
- The next step for this project is to add a cubic term to the potential energy approximation, and see what effect coupling will have on $<\phi^2>$. Preliminary results show the possibility of enhancement of $<\phi^2>$ (see above).

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